Engineering Notes

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Optimal Control Laws for Momentum-Wheel Desaturation Using Magnetorquers

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Introduction

EARTH-pointing satellites are expected to maintain the local-vertical/local-horizontal attitude in the presence of environmental disturbance effects. In many applications this is achieved by using momentum-wheel systems which continuously exchange angular momentum with the spacecraft (SC) body. Because of secular external disturbances, such as the aerodynamic and solar radiation pressure torque, the wheel will accumulate (or lose) angular momentum, drifting towards its maximum (or minimum) allowed speed limits. Hence, suitable control of the wheel's angular momentum is required to counteract the influence of a persisting external disturbance torque. An external control torque must be applied to perform the wheel's momentum dumping and force the angular velocity back into its allowable limits. Magnetic attitude control of spinning and momentum biased SC is one of the earliest techniques employed in attitude stabilization [1–4]. A number of relevant studies on wheel's momentum dumping using magnetorquers are available, both for low earth orbiting (LEO) SC [5–11] and higher altitude satellites [12–14]. Whereas reaction thrusters allow rapid momentum dumping with a large propellant consumption, the use of magnetorquers provides a cheaper desaturation maneuver, without the use of any propellant.

In this Note we derive the control laws for a satellite equipped with magnetic dipoles which allow an optimal momentum-wheel desaturation. To this end, we minimize a cost function taking into account both the maneuvering time and the power consumption. The problem is solved through an indirect approach, and the dependence of the optimal control law on the SC orbital parameters and its characteristics is discussed. Also, the minimum time and the minimum power problem are solved by using a semi-analytical and a closed-form solution, respectively.

A number of numerical simulations based on the proposed methodology have been applied to the ALMASat Microsatellite [15], whose program was established and developed at the University of Bologna in 2003.

Problem Formulation

The dynamic model of an Earth-pointing rigid satellite, using three orthogonal magnetic coils and a momentum-wheel set, in a body axes inertia principal reference frame $\mathcal{T}_B(x, y, z)$ is

$$\mathbb{I} \cdot \dot{\boldsymbol{\omega}} + \boldsymbol{\omega} \times (\mathbb{I} \cdot \boldsymbol{\omega}) + \dot{\boldsymbol{h}} + \boldsymbol{\omega} \times \boldsymbol{h} = \boldsymbol{M}^{(m)} + \boldsymbol{M}^{(d)} \qquad (1)$$

where $[\boldsymbol{\omega}]_{\mathcal{T}_B} = [\omega_x, \omega_y, \omega_z]^T$ is the angular velocity vector, $\mathbb{I}$ is the spacecraft inertia dyadic, $\boldsymbol{M}^{(d)}$ is the total external disturbance torque, $\boldsymbol{M}^{(m)}$ is the control torque due to magnetorquers and $[\boldsymbol{h}]_{\mathcal{T}_B} = [h_x, h_y, h_z]^T$ is the angular momentum vector of the wheel set.

We assume the spacecraft is three-axis stabilized ($\dot{\boldsymbol{\omega}} = 0$) with a perfect nadir pointing during the desaturation process ($\omega_x = \omega_z = 0$). To this end, an additional control law (see, for example, the one implemented in [16]) acts to maintain nominal pointing of the y-axis towards the orbit normal. Thus Eq. (1) gives the following simplified dumping model for the momentum-wheel set [11]:

$$\dot{\boldsymbol{h}} + \boldsymbol{\omega} \times \boldsymbol{h} = \boldsymbol{M}^{(m)} + \boldsymbol{M}^{(d)} \qquad (2)$$

The torque generated by the magnetorquers can be modeled as

$$\boldsymbol{M}^{(m)} = \boldsymbol{m} \times \boldsymbol{B} \qquad (3)$$

where $[\boldsymbol{m}]_{\mathcal{T}_B} = [m_x, m_y, m_z]^T$ is the commanded magnetic dipole moment vector generated by the coils and $[\boldsymbol{B}]_{\mathcal{T}_B} = [B_x, B_y, B_z]^T$ is the local geomagnetic field vector.

In an ideal case ($\boldsymbol{M}^{(d)} = 0$) of a satellite using a y-axis momentum wheel only ($h_x = h_z = 0$), from Eqs. (2) and (3) the wheel's angular momentum dynamics is

$$\dot{h}_y = -m_x B_z + m_z B_x \qquad (4)$$

For a circular low Earth orbit of radius r_c, the geomagnetic field is approximately that of a magnetic dipole [8,11,14,17] of moment $M_\oplus = 7.8379 \times 10^6\ \mathrm{T \cdot km^3}$:

$$[\boldsymbol{B}]_{\mathcal{T}_B} = \frac{M_\oplus}{r_c^3} \begin{bmatrix} \sin i \cos u \\ -\cos i \\ 2 \sin i \sin u \end{bmatrix} \qquad (5)$$

where i is the orbital inclination and $u = \omega_0 t$ is the argument of latitude. Equation (5) quantifies the magnetic field components in the local-level reference frame [18] which is aligned with the body-fixed reference frame $\mathcal{T}_B$ in case of perfect nadir pointing [17,19]. The magnetic dipole model, although approximate and susceptible to the introduction of simulation errors [20], is widely used in the design phase. As a matter of fact, its compact structure allows the designer to obtain analytical or semi-analytical solutions to control problems.

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Optimal Desaturation Control Law

Let $t_0 \in [0, 2\pi\sqrt{r_c^3/\mu_\oplus}]$ ($\mu_\oplus$ is the Earth's gravitational parameter) be the initial time instant. The desaturation maneuver consists of a preselected wheel angular momentum variation in a time interval $\Delta t \triangleq t_1 - t_0$. During the maneuver, the angular momentum variation is given by $\Delta h_y \triangleq h_{y_1} - h_{y_0}$, where $h_{y_0} = h_y(t_0)$, and $h_{y_1} = h_y(t_1)$, with $h_{y_1} < h_{y_0}$.

Our aim is to optimize the maneuver by taking into account suitable constraints on both the time interval, Δt, and the power consumption. The latter is proportional to the cost function J_P defined as [11]

$$J_P \triangleq \frac{1}{2}\int_{t_0}^{t_1} \left(m_x^2 + m_z^2\right) \mathrm{d}t \tag{6}$$

Short maneuvers require a great amount of energy in a shorter time, whereas long maneuvers are prone to the risk of not being completed in full sunlight. Accordingly, the problem addressed here is that of finding the optimal control law $\boldsymbol{u}(t) \triangleq [m_x(t), m_z(t)]^T$ (where $t \in [t_0, t_1]$, $m_x \in [-\bar{m}_x, \bar{m}_x]$, and $m_z \in [-\bar{m}_z, \bar{m}_z]$ with $(\bar{m}_x, \bar{m}_z) > 0$), which maximizes the performance index:

$$J(a) = -a\Delta t - (1-a)J_P = -a\Delta t - \frac{(1-a)}{2}\int_{t_0}^{t_1}\left(m_x^2 + m_z^2\right)\mathrm{d}t \tag{7}$$

where $a \in [0, 1]$ is a positive parameter which allows the designer to give different weights to the maneuvering time and to the power consumption, respectively.

From (4) and (7) the Hamiltonian of the system is [21]

$$H = -\frac{(1-a)}{2}\left(m_x^2 + m_z^2\right) + \lambda(-m_x B_z + m_z B_x) \tag{8}$$

where λ is the adjoint to the wheel's angular momentum h_y. Its time derivative is provided by the Euler–Lagrange equation:

$$\dot{\lambda} = -\frac{\partial H}{\partial h_y} = 0 \tag{9}$$

Equation (9) shows that λ is a constant of motion.

From Pontryagin's maximum principle the optimal control law $\boldsymbol{u}(t)$, to be selected in the domain of feasible controls $\mathcal{U}$, is such that, at any time, the Hamiltonian is an absolute maximum. That is

$$\boldsymbol{u} = \arg\max_{\boldsymbol{u}\in\mathcal{U}} H \tag{10}$$

Assuming $a \neq 1$ and using the necessary condition $\partial H/\partial \boldsymbol{u} = 0$ for interior points of the feasible set $\mathcal{U}$, the following expressions for the onboard magnetic dipoles are obtained:

$$m_x = \begin{cases} \mathrm{sgn}(-\lambda B_z)\bar{m}_x & \text{if } \|\lambda B_z/(1-a)\| > \bar{m}_x \\ -\frac{\lambda B_z}{(1-a)} & \text{if } \|\lambda B_z/(1-a)\| \le \bar{m}_x \end{cases} \tag{11}$$

$$m_z = \begin{cases} \mathrm{sgn}(\lambda B_x)\bar{m}_z & \text{if } \|\lambda B_x/(1-a)\| > \bar{m}_z \\ \frac{\lambda B_x}{(1-a)} & \text{if } \|\lambda B_x/(1-a)\| \le \bar{m}_z \end{cases} \tag{12}$$

where sgn(.) is the signum function. The value of the adjoint variable λ cannot be obtained analytically, but is found by numerically solving a two-point boundary value problem (TPBVP) whose final condition is $h_y(t_1) = h_{y_1}$. Finally, the optimal final time t_1 is obtained by enforcing the transversality condition [21] $H(t_1) = a$.

It is interesting to compare the optimal control law given by Eqs. (11) and (12) with the so-called cross product law (CPL) [1,2,5]:

$$m_x = \begin{cases} m_{x_{\mathrm{CPL}}} \triangleq -k\left(1 - \frac{h_y}{h_{y_1}}\right)\frac{B_z}{M_\oplus/r_c^3} & \text{if } \|m_{x_{\mathrm{CPL}}}\| \le \bar{m}_x \\ \bar{m}_x \mathrm{sgn}(m_{x_{\mathrm{CPL}}}) & \text{if } \|m_{x_{\mathrm{CPL}}}\| > \bar{m}_x \end{cases} \tag{13}$$

$$m_z = \begin{cases} m_{z_{\mathrm{CPL}}} \triangleq k\left(1 - \frac{h_y}{h_{y_1}}\right)\frac{B_x}{M_\oplus/r_c^3} & \text{if } \|m_{z_{\mathrm{CPL}}}\| \le \bar{m}_z \\ \bar{m}_z \mathrm{sgn}(m_{z_{\mathrm{CPL}}}) & \text{if } \|m_{z_{\mathrm{CPL}}}\| > \bar{m}_z \end{cases} \tag{14}$$

where $k > 0$ is a control gain. Under the previous assumption ($h_y/h_{y_1} > 1$), an infinitely large control gain ($k \to \infty$) approaches the bang-bang law given by Eqs. (11) and (12) in the case of saturated coils.

Clearly, due to its open loop nature, the optimal control laws expressed by Eqs. (11) and (12) is sensitive to unexpected disturbances and modeling errors.

Nevertheless, it provides the global maximum of $J(a)$ [see Eq. (7)] and, as such, it can be used effectively as a reference value when the performances of closed-loop feedback laws are investigated.

Moreover, whereas the determination of λ for a TPBVP is usually a critical issue [21,22], in this case a suitable first guess value of the adjoint is obtained from the analytical solution of the problem (10) in the case of minimum time ($a = 1$) and minimum energy consumption ($a = 0$).

Minimum Desaturation Time

Assuming $a = 1$ in Eq. (7), we are faced with the problem of finding the minimum time interval, $\Delta t^\star = \min(\Delta t)$, required for a given angular momentum variation Δh_y. This corresponds to a performance index $J(1) = -\Delta t$.

From Eq. (8) it turns out that $H(a = 1)$ depends linearly on the controls m_x and m_z. As a result, a bang-bang control is optimal [23]:

$$m_x = -\mathrm{sgn}(\lambda B_z)\bar{m}_x \tag{15}$$

$$m_z = \mathrm{sgn}(\lambda B_x)\bar{m}_z \tag{16}$$

Note that this problem provides a general semi-analytical solution, and that a TPBVP solution is not necessary. When Eqs. (15) and (16) are substituted into Eq. (4) one has

$$\dot{h}_y = \mathrm{sgn}(\lambda)[\bar{m}_x\|B_z\| + \bar{m}_z\|B_x\|] \tag{17}$$

Because the term between square brackets is positive, one obtains

$$\mathrm{sgn}\,(\lambda) = \mathrm{sgn}(\Delta h_y) = -1 \tag{18}$$

When Eqs. (18) and (5) are substituted into Eq. (17), the following result is found:

$$\widetilde{\Delta h_y} \triangleq \frac{-\Delta h_y}{(\bar{m}_x M_\oplus \|\sin i\|)/(\omega_0 r_c^3)} = \int_{u_0}^{u_1}\left(2\|\sin u\| + \frac{\bar{m}_z}{\bar{m}_x}\|\cos u\|\right)\mathrm{d}u \tag{19}$$

where $u_0 \triangleq \omega_0 t_0$ and $u_1 \triangleq \omega_0 t_1 = \omega_0 t_0 + \omega_0 \Delta t^\star$ are the initial and final argument of latitude, respectively.

If it is assumed that u_0 and $\bar{m}_z/\bar{m}_x$ are given, the solution of the integral (19) with respect to u_1 provides the minimum time interval $\Delta t^\star$ required to obtain the dimensionless angular momentum variation $\widetilde{\Delta h_y}$. Figure 1 shows the variations of the dimensionless angular momentum $\widetilde{\Delta h_y}$ as a function of $\Delta t^\star$, and u_0 when $\bar{m}_z/\bar{m}_x = 1$. Figure 1 also shows that, for small values of the desaturation interval, the corresponding values of $\widetilde{\Delta h_y}$ may vary widely with u_0. This behavior tends to be smoothed when longer maneuvers are considered. As a matter of fact, in those cases the achievable desaturation interval tends to become constant with u_0. Figure 1, with an appropriate scale factor, can be used to compute the energy required for performing the maneuver. Indeed, substituting Eqs. (15) and (16) into Eq. (6) yields

$$J_P(\Delta t^\star) = \frac{\bar{m}_x^2 + \bar{m}_z^2}{2}\Delta t^\star \tag{20}$$

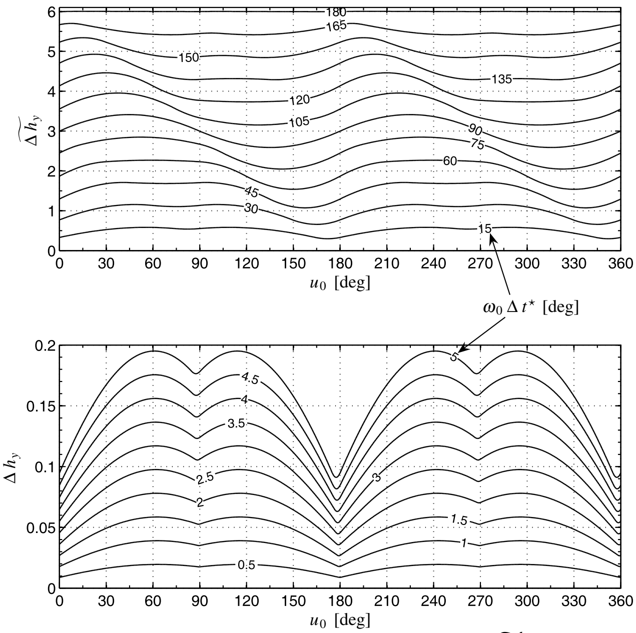

Fig. 1 Minimum time interval $\Delta t^\star$ as a function of $\widetilde{\Delta h}_y$ and u_0 for $\bar{m}_x = \bar{m}_z$.

Minimum Desaturation Energy

The power consumption problem can be addressed by selecting $a = 0$ in Eq. (7). The internal dipoles time history is still given by Eqs. (11) and (12), but an additional constraint on the time interval Δt must now be imposed. Actually, without a suitable constraint on the terminating time t_1, the solution to the optimal problem would provide an infinite desaturation time interval. Once Δt is given, the adjoint variable λ is obtained by solving the associated TPBVP and by enforcing the condition $h_y(t_1) = h_{y_1}$.

In an ideal case in which the dipoles are unbounded, a fully analytical solution of the optimal problem can be found. For ideal actuators [that is, $(\bar{m}_x, \bar{m}_z) \to \infty$], the control equations are [see Eqs. (11) and (12)]

$$[m_x, m_z]^T = \lambda[-B_z, B_x]^T \tag{21}$$

When Eq. (21) is substituted into Eq. (4) one obtains

$$\dot{h}_y = \lambda\left(B_x^2 + B_z^2\right) \tag{22}$$

and, introducing Eq. (5) into (22) yields

$$\dot{h}_y = A[1 + 3\sin^2(\omega_0 t)] \tag{23}$$

where

$$A \triangleq \lambda \frac{M_\oplus^2 \sin^2 i}{r_c^6} \tag{24}$$

Note that A is a constant, provided that the orbital perturbations are neglected during the maneuver. Solving the differential Eq. (23), the following time history of the wheel's angular momentum is found:

$$h_y(t) = \frac{5}{2}A(t - t_0) + \frac{3A[\sin(2\omega_0 t_0) - \sin(2\omega_0 t)]}{4\omega_0} + h_{y_0} \tag{25}$$

Enforcing the final time condition, $h_y(t_1) = h_{y_1}$, one obtains

$$\lambda = \frac{4 r_c^6 \omega_0 (h_{y_1} - h_{y_0})}{M_\oplus^2 \sin^2 i[10\omega_0 \Delta t + 3\sin(2\omega_0 t_0) - 3\sin(2\omega_0 t_0 + 2\omega_0 \Delta t)]} \tag{26}$$

Note that the value of λ, which solves the optimal problem, depends (through the initial time t_0) on the orbital point where the maneuver starts. Also, λ depends on the time interval Δt and the required desaturation interval Δh_y. As previously discussed, the λ value of Eq. (26) can be used as an initial guess solution for a TPBV problem with $a \neq 0$.

The value of the initial time $t_0^\star$ corresponding to the minimum power consumption is found observing that the energy is a function of t_0 through the term $\|\lambda\|$. Substituting Eqs. (21) and (22) into (7) yields

$$J(0) = -\frac{\|\lambda\|}{2}\int_{t_0}^{t_1} \|\dot{h}_y\| \, \mathrm{d}t = -\frac{\|\lambda\|}{2}\|\Delta h_y\| \tag{27}$$

From Eq. (27), the value of $t_0^\star$ coincides with the value that minimizes the expression of $\|\lambda\|$ given by Eq. (26):

$$t_0^\star = \arg\max_{t_0} J(0) = \frac{1}{2\omega_0}\arctan\left[\frac{\cos(2\omega_0\Delta t) - 1}{\sin(2\omega_0 \Delta t)}\right] \tag{28}$$

A final remark is in order. The optimal control law given by Eqs. (21) coincides with the (open-loop) minimum energy controller found by Chen et al. [11], when a single momentum wheel is considered. However, in the general case of three momentum wheels, there are three adjoint variables whose initial values cannot be found analytically. Therefore, the closed-form solution is not available and the TPBVP must be solved numerically. Note, instead, that as long as a single momentum wheel is considered, an analytical solution does exist and is given by Eq. (26).

Case Study

The control laws described in the preceding section have been applied to study an optimal desaturation maneuver for the ALMASat spacecraft [15]. ALMASat is a three-axis stabilized, multipurpose student microsatellite platform, which could be used with minor modifications for a number of scientific missions. The spacecraft attitude control is based on three orthogonal magnetic coils ($\bar{m}_x = \bar{m}_y = \bar{m}_z$) for attitude acquisition maneuvers and for coarse attitude pointing, and a pitch momentum wheel with moment of inertia I_w. ALMASat main parameters [15] are summarized in Table 1.

The desaturation maneuver consists of a momentum-wheel velocity variation from an initial value $\omega_{w_0} \triangleq \omega_w(t_0) = 7350$ rpm ($h_{y_0} = 35.2518$ mN · m · s) to a final value $\omega_{w_1} \triangleq \omega_w(t_1) =$ 4700 rpm ($h_{y_1} = 22.5420$ mN · m · s). The corresponding variation of the angular momentum is $\Delta h_y = -12.7098$ mN · m · s [$\widetilde{\Delta h}_y =$ 0.415348, as stated in Eq. (19)].

Figure 2 shows the minimum maneuvering time $\Delta t^\star$ as a function of the maneuver starting point along the orbit (u_0). Figure 2 also shows that for the preselected Δh_y, the minimum time is found in the time interval $\Delta t^\star \in [173.3, 330.4]$ s. The global minimum value of $\Delta t^\star$ (with respect to u_0) is found at four points along the orbit, where $u = (58, 111, 238, 291)$ deg. The maximum value of $\Delta t^\star$ occurs for maneuvers starting when the spacecraft lies nearly in the Earth's equatorial plane, $u = (170, 350)$ deg. According to Eq. (20), the cost function related to the power consumption is proportional to $\Delta t^\star$, $J_P \in [443.648, 845.824]$ $A^2 \cdot m^4 \cdot s$. Smaller values of J_P can be obtained using the cost function $J(a)$ and a value of $a \neq 1$ [recall from Eq. (7) that $a = 1$ corresponds to the minimum time solution]. This choice corresponds to less expensive (in terms of power consumption) but longer maneuvers.

Choosing $a = 0.5$ (which corresponds to the same weight for both J_P and Δt) and $u_0 = 58$ deg, we obtain $\Delta t \simeq 341.8$ s which is about

Table 1 ALMASat main parameters

Parameter	Value
Momentum-wheel inertia (I_w)	4.58×10^{-5} kg · m^2
Maximum magnetic dipole ($\bar{m}_x = \bar{m}_z$)	1.6 A · m^2
Orbit radius (r_c)	7021 km
Inclination (i)	65 deg

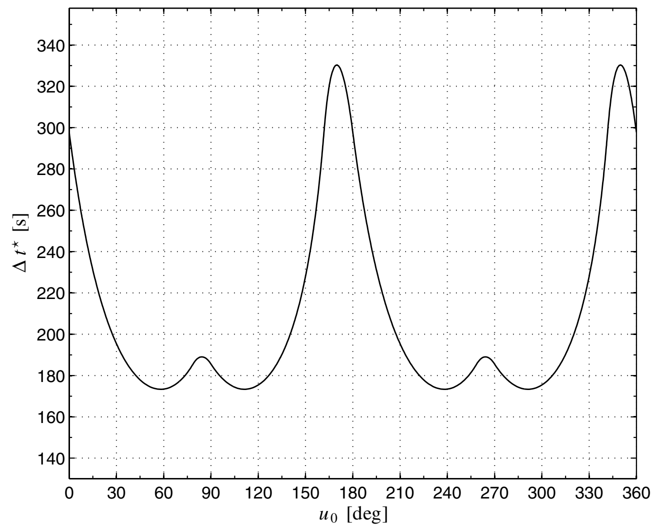

Fig. 2 Minimum time interval $\Delta t^{\star}$ as a function of u_0 for a desaturation $\Delta h_y = -12.7098$ mN · m · s.

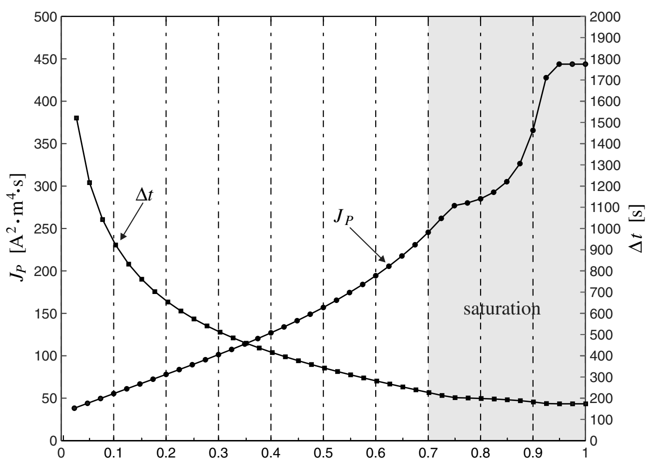

Fig. 3 J_P and Δt values as a function of a ($u_0 = 58$ deg).

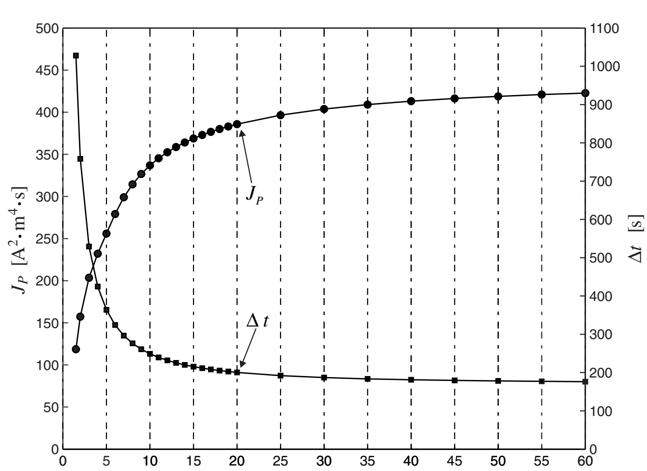

Fig. 4 CPL performances as a function of gain k ($u_0 = 58$ deg).

twice the minimum time corresponding to the selected value of u_0. However, albeit at the price of increasing the maneuvering time, we obtain a reduction in the cost function, $J_P = 156.96$ $A^2 \cdot m^4 \cdot s$, which is more than two times lower than the value previously obtained in the case of minimum time maneuver $\min[J_P(\Delta t^{\star})] = 443.648$ $A^2 \cdot m^4 \cdot s$. By varying the parameter a, several combinations can be obtained, thus leading to a tradeoff between desaturation time (Δt) and energy required for the maneuver (J_P). This is clearly shown in Fig. 3.

Figure 3 also shows that for $a = 0.5$ and $u_0 = 58$ deg, the internal magnetic dipoles are not saturated during the whole maneuver. Note that the adjoint variable value $\lambda \simeq -24699.34$ $A \cdot m^2/T$, given by the solution of the TPBVP, coincides with the value obtainable by substituting both $\Delta t \simeq 341.8$ s (resulting from the TPBVP) and $u_0 = \omega_0 t_0 = 58$ deg into Eq. (26).

Figure 4 shows the performances of the CPL related to the same maneuver as a function of the gain k appearing in Eqs. (13) and (14). When a desaturation time $\Delta t \simeq 341.8$ s (corresponding to the optimal control law) is selected, the energy required by the CPL is $J_P = 268.5$ $A^2 \cdot m^4 \cdot s$, with an increase of 71% with respect the optimal value. An additional comparison can be made with the LQR feedback law presented in [11]. To make a meaningful comparison, the LQR weights have been tailored to guarantee that the settling time be equal to 341.8 s. In this case the energy required is $J_P = 522.8$ $A^2 \cdot m^4 \cdot s$, almost three times larger than the optimal value.

Conclusions

The problem of optimal reaction wheel desaturation maneuver of a satellite using internal magnetorquers has been discussed. Minimum time and minimum power maneuvers are performed by selecting appropriate weights for the coefficients of the proposed cost function. The time histories for internal dipoles have been obtained using a semi-analytical solution, which takes into account the upper and lower limits of the actuators. In case of ideal actuators, the relationships between optimal control laws and orbital parameters are expressed in closed form. For a given desaturation interval, this allows to identify the point along the orbit where the maneuver should start.

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